Analízis gyakorló feladatok - megoldás
Meteorológus MSc, 2011/2012. I. félév

1. Konvergensek-e a következő sorozatok? Ha igen, mi a határértékük?

(a) $a_n = \frac{1}{n} \to 0;$

(b) $a_n = \sqrt[n]{n} \to 1;$

(c) $a_n = \sqrt[n]{a} \to 1 \ (a > 0);$

(d) $a_n = q^n \to 0$, ha $|q| < 1;$

(e) $a_n = n^k \cdot q^n \to 0$, ha $|q| < 1;$

(f) $a_n = \frac{n^k}{c^n} \to 0$, ha $c > 1;$

(g) $a_n = \frac{c^n}{n!} \to 0;$

(h) $a_n = \frac{n!}{n^n} \to 0;$

(i) $a_n = \sqrt{n+1} - \sqrt{n} = \frac{(\sqrt{n+1}-\sqrt{n})(\sqrt{n+1}+\sqrt{n})}{\sqrt{n+1}+\sqrt{n}} = \frac{1}{\sqrt{n+1}+\sqrt{n}} \to 0;$

(j) $a_n = \frac{1}{n^2} \sum_{k=1}^{n} k = \frac{1}{n^2} \frac{n(n+1)}{2} \to \frac{1}{2};$

(k) $a_n = \frac{2^{2n}+n^2}{5^n-n} = \frac{\left(\frac{4}{5}\right)^n + \frac{n^2}{5^n}}{1-\frac{n}{5^n}} \to \frac{0+0}{1+0} = 0;$

(l) $a_n = \frac{3 \cdot 8^{2n} - n^{10} \cdot 3^{3n}}{n^2 \cdot 5^n - 2 \cdot 4^{3n+1}} = \frac{3 \cdot \left(\frac{8^2}{4^3}\right)^n - n^{10} \cdot \frac{3^{3n}}{4^{3n}}}{n^2 \cdot \frac{5^n}{4^{3n}} - 2 \cdot 4} \to \frac{3-0}{0-8} = -\frac{3}{8};$

(m) $a_n = \sqrt[n]{b_n} \to 1$, ha $b_n \to a > 0;$

(n) $a_n = \sqrt[n+2]{2n+3} = \sqrt[n+2]{n} \cdot \sqrt[n+2]{2 + \frac{3}{n}} \to 1 \cdot 1 = 1;$

(o) $a_n = \left(1 + \frac{1}{n}\right)^{2n+3} = \left(\left(1 + \frac{1}{n}\right)^n\right)^2 \cdot \left(1 + \frac{1}{n}\right)^3 \to \mathrm{e}^2 \cdot 1 = \mathrm{e}^2;$

(p) $a_n = \left(1 + \frac{1}{n^3}\right)^{n^2} = \sqrt[n]{\left(1 + \frac{1}{n^3}\right)^{n^3}} \to 1;$

(q) $a_n = \sqrt[n]{5n+3} = \sqrt[n]{n} \cdot \sqrt[n]{5 + \frac{3}{n}} \to 1 \cdot 1 = 1;$

(r) $a_n = \sqrt[2n]{n} = \sqrt{\sqrt[n]{n}} \to \sqrt{1} = 1;$

(s) $a_n = \sqrt[n]{2 + \sqrt{n}} = \sqrt{\sqrt[n]{n}} \cdot \sqrt[n]{\frac{2}{\sqrt{n}} + 1} \to \sqrt{1} \cdot 1 = 1;$

(t) $a_n = \sqrt[n]{10n^3 + 2} = (\sqrt[n]{n})^3 \cdot \sqrt[n]{10 + \frac{2}{n^3}} \to 1^3 \cdot 1 = 1;$

(u) $a_n = \sqrt[n]{2^n + n^3} = 2 \cdot \sqrt[n]{1 + \frac{n^3}{2^n}} \to 2 \cdot 1 = 2;$

(v) $a_n = \sqrt[n]{\frac{5^n+3^n}{4^n-3^n}} = \frac{5 \cdot \sqrt[n]{1+\left(\frac{3}{5}\right)^n}}{4 \cdot \sqrt[n]{1-\left(\frac{3}{4}\right)^n}} \to \frac{5 \cdot 1}{4 \cdot 1} = \frac{5}{4};$

(w) $a_n = \sqrt[2n]{\frac{n^2 \cdot 2^n + 3^n}{\sqrt{4^n - 3^n}}} = \frac{\sqrt{3 \cdot \sqrt[n]{n^2 \cdot \left(\frac{2}{3}\right)^n + 1}}}{\sqrt[4]{4 \cdot \sqrt[n]{1-\left(\frac{3}{4}\right)^n}}} \to \frac{\sqrt{3 \cdot 1}}{\sqrt[4]{4 \cdot 1}} = \frac{\sqrt{3}}{\sqrt[4]{4}};$

(x) $a_n = \sqrt[4n]{4^n + n^4 + n^3} = \sqrt[4]{4} \cdot \sqrt[4]{\sqrt[n]{1 + \frac{n^4}{4^n} + \frac{n^3}{4^n}}} \to \sqrt[4]{4} \cdot \sqrt[4]{1} = \sqrt[4]{4}.$

2. Határozzuk meg az alábbi sorozatok határértékét (ha az létezik)!

(a) $a_n = \frac{3n+5}{7n-8} = \frac{3+\frac{5}{n}}{7-\frac{8}{n}} \to \frac{3}{7};$

(b) $a_n = \frac{50n^2+25n}{n^3+1} = \frac{\frac{50}{n}+\frac{25}{n^2}}{1+\frac{1}{n^3}} \to \frac{0}{1} = 0;$

(c) $a_n = \frac{4n^5+n-2}{8n^3+7n^2+1} = \frac{4n^2+\frac{1}{n^2}-\frac{2}{n^3}}{8+\frac{7}{n}+\frac{1}{n^3}} \to \frac{\infty}{8} = \infty;$

(d) $a_n = \frac{3^n+2^n}{4\cdot 3^n-2\cdot 2^n} = \frac{1+\left(\frac{2}{3}\right)^n}{4-2\cdot\left(\frac{2}{3}\right)^n} \to \frac{1}{4};$

(e) $a_n = \frac{\frac{5}{n^4}+\frac{2}{n^2}}{\frac{8}{n^5}+\frac{1}{n^3}+\frac{1}{n^2}} = \frac{\frac{5}{n^2}+2}{\frac{8}{n^3}+\frac{1}{n}+1} \to \frac{2}{1} = 2;$

(f) $a_n = \frac{\sqrt[n]{125}-1}{\sqrt[n]{5}-1} = \frac{(\sqrt[n]{5})^3-1^3}{\sqrt[n]{5}-1} = (\sqrt[n]{5})^2+\sqrt[n]{5}+1 \to 1^2+1+1 = 3;$

(g) $a_n = \sqrt[n]{5^n+2^n} = 5\cdot\sqrt[n]{1+\left(\frac{2}{5}\right)^n} \to 5\cdot 1 = 5;$

(h) $a_n = \sqrt[n]{5^n-2^n} = 5\cdot\sqrt[n]{1-\left(\frac{2}{5}\right)^n} \to 5\cdot 1 = 5;$

(i) $a_n = \left(1+\frac{2}{n}\right)^n \to \mathrm{e}^2;$

(j) $a_n = \left(1+\frac{1}{3n}\right)^n \to \mathrm{e}^{1/3} = \sqrt[3]{\mathrm{e}};$

(k) $a_n = \left(\frac{n-1}{n+2}\right)^n = \left(1-\frac{3}{n+2}\right)^{n+2}\cdot\left(1-\frac{3}{n+2}\right)^{-2} \to \mathrm{e}^{-3}\cdot 1^{-2} = \frac{1}{\mathrm{e}^3};$

(l) $a_n = \prod_{k=2}^n\left(1-\frac{1}{k}\right) = \left(1-\frac{1}{2}\right)\cdot\left(1-\frac{1}{3}\right)\cdots\left(1-\frac{1}{n}\right) = \frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdots\frac{n-1}{n} = \frac{1}{n} \to 0.$

3. Döntsük el, hogy léteznek-e az alábbi függvényhatárértékek, és ha igen, akkor számítsuk ki ezeket!

(a) $\lim_{x\to 1}\frac{x^2-x}{x^3-x} = \lim_{x\to 1}\frac{(x-1)x}{(x-1)(x^2+x)} = \lim_{x\to 1}\frac{x}{x^2+x} = \frac{1}{2};$

(b) $\lim_{x\to 1}\frac{x^2-1}{2x^2-x-1} = \lim_{x\to 1}\frac{(x-1)(x+1)}{(x-1)(2x+1)} = \lim_{x\to 1}\frac{x+1}{2x+1} = \frac{2}{3};$

(c) $\lim_{x\to 3}\frac{x^2-7x+12}{2x^2-5x-3} = \lim_{x\to 3}\frac{(x-3)(x-4)}{(x-3)(2x+1)} = \lim_{x\to 3}\frac{x-4}{2x+1} = -\frac{1}{7};$

(d) $\lim_{x\to 1}\frac{5}{x^5-1}-\frac{3}{x^3-1} = \lim_{x\to 1}\frac{5(x^3-1)-3(x^5-1)}{(x^5-1)(x^3-1)} = \lim_{x\to 1}\frac{(x-1)^2(-3x^3-6x^2-4x-2)}{(x-1)(x^4+x^3+x^2+x+1)(x-1)(x^2+x+1)} = \lim_{x\to 1}\frac{-3x^3-6x^2-4x-2}{(x^4+x^3+x^2+x+1)(x^2+x+1)} = -\frac{15}{15} = -1;$

(e) $\lim_{x\to 1}\frac{\sqrt{x-1}}{\sqrt[3]{x^2-1}} = \lim_{x\to 1}\sqrt[6]{\frac{(x-1)^3}{(x-1)^2(x+1)^2}} = \lim_{x\to 1}\sqrt[6]{\frac{x-1}{(x+1)^2}} = \sqrt[6]{\frac{0}{4}} = 0;$

(f) $\lim_{x\to -2}\frac{\sqrt[3]{x-6}+2}{x^3+8} = \lim_{x\to -2}\frac{\frac{x+2}{(\sqrt[3]{x-6})^2-2\sqrt[3]{x-6}+4}}{(x+2)(x^2-2x+4)} =$

$= \lim_{x\to -2}\frac{1}{((\sqrt[3]{x-6})^2-2\sqrt[3]{x-6}+4)(x^2-2x+4)} = \frac{1}{12\cdot 12} = \frac{1}{144};$

(g) $\lim_{x\to 1}\frac{1}{x-1}$ nem létezik, mert $\lim_{x\to 1-0}\frac{1}{x-1} = -\infty \neq \lim_{x\to 1+0}\frac{1}{x-1} = \infty;$

(h) $\lim_{x\to 0}\frac{e^x}{x^2} = \frac{1}{0+} = \infty;$

(i) $\lim_{x\to 2}\frac{2+\sqrt{x}}{x^3-8}$ nem létezik, mert $\lim_{x\to 2-0}\frac{2+\sqrt{x}}{x^3-8} = -\infty \neq \lim_{x\to 2+0}\frac{2+\sqrt{x}}{x^3-8} = \infty;$

(j) $\lim_{x\to 0} x^2 \cdot \operatorname{sgn} x = 0$ (sgn függvény korlátos);

(k) $\lim_{x\to 0} \frac{\operatorname{sgn} x}{x^3} = \lim_{x\to 0} \frac{1}{|x|^3} = \frac{1}{0+} = \infty$;

(l) $\lim_{x\to 0} e^{x^2} \cdot \operatorname{sgn} x$ nem létezik, mert $\lim_{x\to 0-0} e^{x^2} \cdot \operatorname{sgn} x = -1 \neq \lim_{x\to 0+0} e^{x^2} \cdot \operatorname{sgn} x = 1$;

(m) $\lim_{x\to\infty} x^2 - 2x - 10 = \lim_{x\to\infty} x(x-2) - 10 = \infty \cdot \infty - 10 = \infty$;

(n) $\lim_{x\to-\infty} x^4 - 3x^3 - 6 = \infty + 3\infty - 6 = \infty$;

(o) $\lim_{x\to-\infty} x - 3\sqrt[3]{x^2} = -\infty - 3\infty = -\infty$;

(p) $\lim_{x\to\infty} \frac{x^2 - x + 4}{x^3 + x + 5} = \lim_{x\to\infty} \frac{\frac{1}{x} - \frac{1}{x^2} + \frac{4}{x^3}}{1 + \frac{1}{x^2} + \frac{5}{x^3}} = \frac{0}{1} = 0$;

(q) $\lim_{x\to-\infty} \frac{x^2 - 5x + 9}{x + 6} = \lim_{x\to-\infty} \frac{x - 5 + \frac{9}{x}}{1 + \frac{6}{x}} = \frac{-\infty - 5 + 0}{1 + 0} = -\infty$;

(r) $\lim_{x\to\infty} \sqrt{x + \sqrt{x}} - \sqrt{x} = \lim_{x\to\infty} \frac{(\sqrt{x + \sqrt{x}} - \sqrt{x})(\sqrt{x + \sqrt{x}} + \sqrt{x})}{\sqrt{x + \sqrt{x}} + \sqrt{x}} = \lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x + \sqrt{x}} + \sqrt{x}} =$
$\lim_{x\to\infty} \frac{1}{\sqrt{1 + \frac{1}{\sqrt{x}}} + 1} = \frac{1}{2}$.

4. A $\lim_{x\to 0} \frac{\sin x}{x} = 1$ azonosság felhasználásával számítsuk ki az alábbi határértékeket, ha azok léteznek!

(a) $\lim_{x\to 0} \frac{\sin x \cos x}{x} = \lim_{x\to 0} \frac{\sin x}{x} \cos x = 1 \cdot 1 = 1$;

(b) $\lim_{x\to 0} \frac{\sin^2 x}{x^2} = \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2 = 1^2 = 1$;

(c) $\lim_{x\to 0} \frac{\sin^2 x}{x^3} = \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2 \cdot \frac{1}{x}$ nem létezik

(d) $\lim_{x\to 0} \frac{1 - \cos x}{x^2} = \lim_{x\to 0} \frac{(1 - \cos x)(1 + \cos x)}{x^2(1 + \cos x)} = \lim_{x\to 0} \frac{1 - \cos^2 x}{x^2(1 + \cos x)} = \lim_{x\to 0} \frac{\sin^2 x}{x^2} \frac{1}{1 + \cos x} =$
$1^2 \cdot \frac{1}{2} = \frac{1}{2}$;

(e) $\lim_{x\to 0} x \cdot \operatorname{ctg} 2x = \lim_{x\to 0} x \cdot \frac{\cos 2x}{\sin 2x} = \lim_{x\to 0} \frac{1}{2} \cdot \frac{2x}{\sin 2x} \cdot \cos 2x = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$;

(f) $\lim_{x\to 0} \operatorname{tg} x \cdot \ln|x| = \lim_{x\to 0} \frac{\sin x}{\cos x} \cdot \ln|x| = \lim_{x\to 0} \frac{\sin x}{x} \cdot (x \ln|x|) \cdot \cos x = 1 \cdot 0 \cdot 1 = 0$;

(g) $\lim_{x\to 0} \frac{1 - \sqrt{\cos x}}{\sin^2 x} = \lim_{x\to 0} \frac{(1 - \sqrt{\cos x})(1 + \sqrt{\cos x})}{\sin^2 x(1 + \sqrt{\cos x})} = \lim_{x\to 0} \frac{1 - \cos x}{x^2} \cdot \frac{x^2}{\sin^2 x} \cdot \frac{1}{1 + \sqrt{\cos x}} =$
$\frac{1}{2} \cdot 1^2 \cdot \frac{1}{2} = \frac{1}{4}$ (ld. (d) pont);

(h) $\lim_{x\to 0} \frac{\operatorname{tg} x - \sin x}{x^3} = \lim_{x\to 0} \frac{\frac{\sin x}{\cos x} - \sin x}{x^3} = \lim_{x\to 0} \frac{\sin x(1 - \cos x)}{\cos x \cdot x^3} = \lim_{x\to 0} \frac{1}{\cos x} \cdot \frac{\sin x}{x} \cdot \frac{1 - \cos x}{x^2} =$
$1 \cdot 1 \cdot \frac{1}{2} = \frac{1}{2}$ (ld. (d) pont).

5. A kompozíciófüggvények deriválására vonatkozó azonosságot is felhasználva számítsuk ki az alábbi hozzárendeléssel adott függvények deriváltjait!

(a1) $f(x) = \sin^5 x$
$f'(x) = 5\sin^4 x \cos x$;

(a2) $f(x) = \sin 5x$
$f'(x) = 5\cos 5x$;

(a3) $f(x) = \sin x^5$
$f'(x) = \cos x^5 \cdot 5x^4$;

(b1) $f(x) = \sin^5 5x^5$
$f'(x) = 5\sin^4 5x^5 \cdot \cos 5x^5 \cdot 25x^4$;

(b2) $f(x) = \sqrt{\frac{1-x}{1+x}}$
$f'(x) = \frac{-1}{\sqrt{1-x}(1+x)^{3/2}}$;

(b3) $f(x) = \frac{x}{\sqrt{1-x^2}}$
$f'(x) = \frac{1}{(1-x^2)^{3/2}}$;

(c1) $f(x) = \sqrt{x+\sqrt{x}}$
$f'(x) = \frac{2\sqrt{x}+1}{4\sqrt{x^2+x\sqrt{x}}}$;

(c2) $f(x) = \frac{1}{(1+x^2)\sqrt{1+x^2}}$
$f'(x) = \frac{-3x}{(1+x^2)^{5/2}}$;

(c3) $f(x) = \sqrt[3]{(1-x)^2}$
$f'(x) = \frac{-2}{3\sqrt[3]{1-x}}$;

(d1) $f(x) = \ln\sqrt{\frac{1-x}{1+x}}$
$f'(x) = \frac{-1}{1-x^2}$;

(d2) $f(x) = \ln\frac{\sqrt{1-x^2}+1}{x}$
$f'(x) = \frac{-1}{x\sqrt{1-x^2}}$;

(d3) $f(x) = \mathrm{e}^{10}\sin 5x$
$f'(x) = \mathrm{e}^{10} 5\cos 5x$;

(e1) $f(x) = \ln\sqrt{\frac{1-\sin x}{1+\sin x}}$
$f'(x) = \frac{-1}{\cos x}$;

(e2) $f(x) = \ln\sin x$
$f'(x) = \operatorname{ctg} x$;

(e3) $f(x) = \ln\ln x$
$f'(x) = \frac{1}{x\ln x}$

(f1) $f(x) = \ln\operatorname{tg}(\frac{x}{2}+\frac{\pi}{4})$
$f'(x) = \frac{2}{\sin(x+\frac{\pi}{2})} = \frac{2}{\cos x}$;

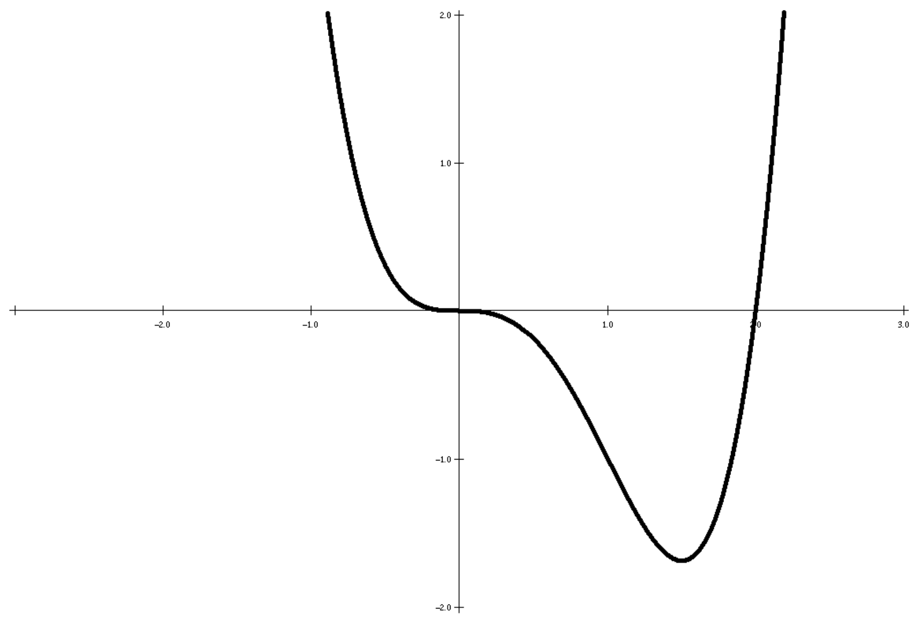
(f2) $f(x) = \ln(x+\sqrt{x^2+a^2})$
$f'(x) = \frac{1}{\sqrt{x^2+a^2}}$.

(f3)

6. Végezzük el az alábbi függvények teljes körű vizsgálatát!

(a) $f(x) = x^4 - 2x^3$

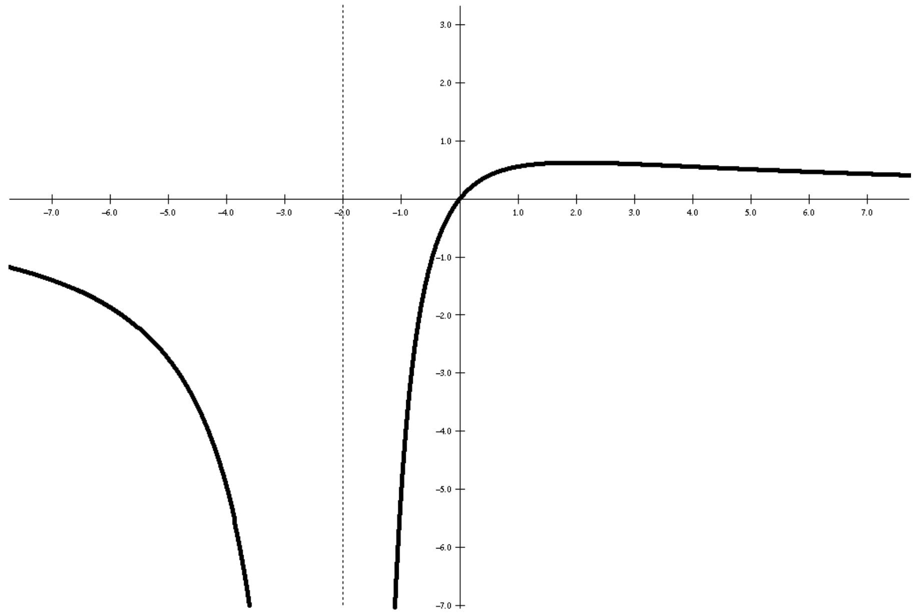
$\mathcal{D}(f) =$	$\mathbb{R}$
$f'(x) =$	$4x^3 - 6x^2$
$f''(x) =$	$12x^2 - 12x$
határértékek:	$\lim_{+\infty} f = \lim_{-\infty} f = +\infty$
monoton nő:	$\left(\frac{3}{2}, +\infty\right)$
monoton fogy:	$\left(-\infty, \frac{3}{2}\right)$
szélsőérték:	$x = \frac{3}{2}$ abszolút minimum
konvex:	$(-\infty, 0) \cup (1, +\infty)$
konkáv:	$(0, 1)$
inflexiós pontok:	$x = 0,\ x = 1$
$\mathcal{R}(f) =$	$[-1,6875; \infty)$



1. ábra. 6.(a)

(b) $f(x) = \frac{5x}{(x+2)^2}$

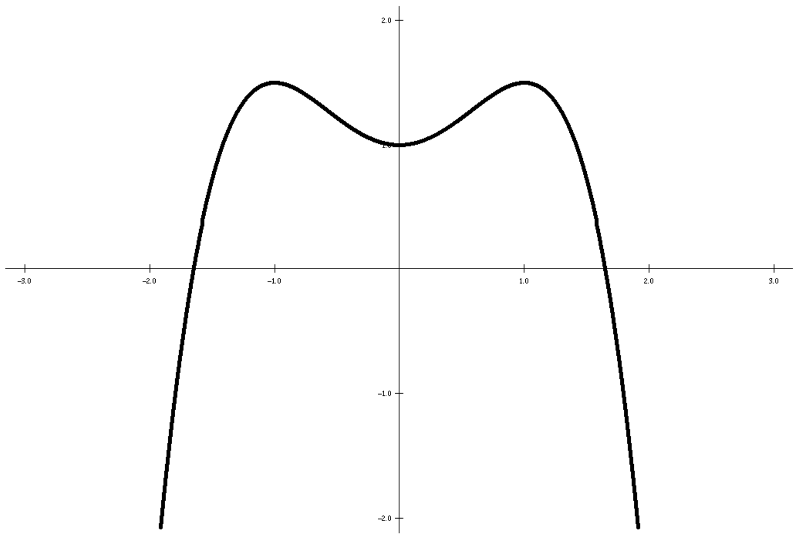
$\mathcal{D}(f) =$	$\mathbb{R} \setminus \{-2\}$
$f'(x) =$	$\frac{-5x+10}{(x+2)^3}$
$f''(x) =$	$\frac{10x-40}{(x+2)^4}$
határértékek:	$\lim_{-2} f = -\infty, \lim_{+\infty} f = \lim_{-\infty} f = 0$
monoton nő:	$(-2, 2)$
monoton fogy:	$(-\infty, -2) \cup (2, +\infty)$
szélsőérték:	$x = 2$ abszolút maximum
konvex:	$(4, +\infty)$
konkáv:	$(-\infty, -2) \cup (-2, 4)$
inflexiós pont:	$x = 4$
$\mathcal{R}(f) =$	$(-\infty, \frac{5}{8}]$



2. ábra. 6.(b)

(c) $f(x) = 1 + x^2 - \frac{x^4}{2}$

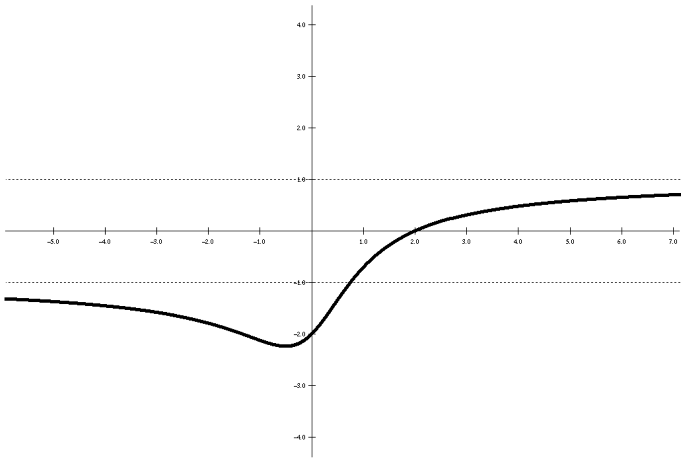
$\mathcal{D}(f) =$	$\mathbb{R}$
$f'(x) =$	$2x - 2x^3$
$f''(x) =$	$2 - 6x^2$
határértékek:	$\lim_{+\infty} f = \lim_{-\infty} f = -\infty$
monoton nő:	$(-\infty, -1) \cup (0, 1)$
monoton fogy:	$(-1, 0) \cup (1, +\infty)$
szélsőértékek:	$x = -1$, $x = 1$ abszolút maximumok,
	$x = 0$ lokális minimum
konvex:	$(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$
konkáv:	$(-\infty, -\frac{1}{\sqrt{3}}) \cup (\frac{1}{\sqrt{3}}, +\infty)$
inflexiós pontok:	$x = -\frac{1}{\sqrt{3}}$, $x = \frac{1}{\sqrt{3}}$
$\mathcal{R}(f) =$	$(-\infty, \frac{3}{2}]$



3. ábra. 6.(c)

(d) $f(x) = \frac{x-2}{\sqrt{x^2+1}}$

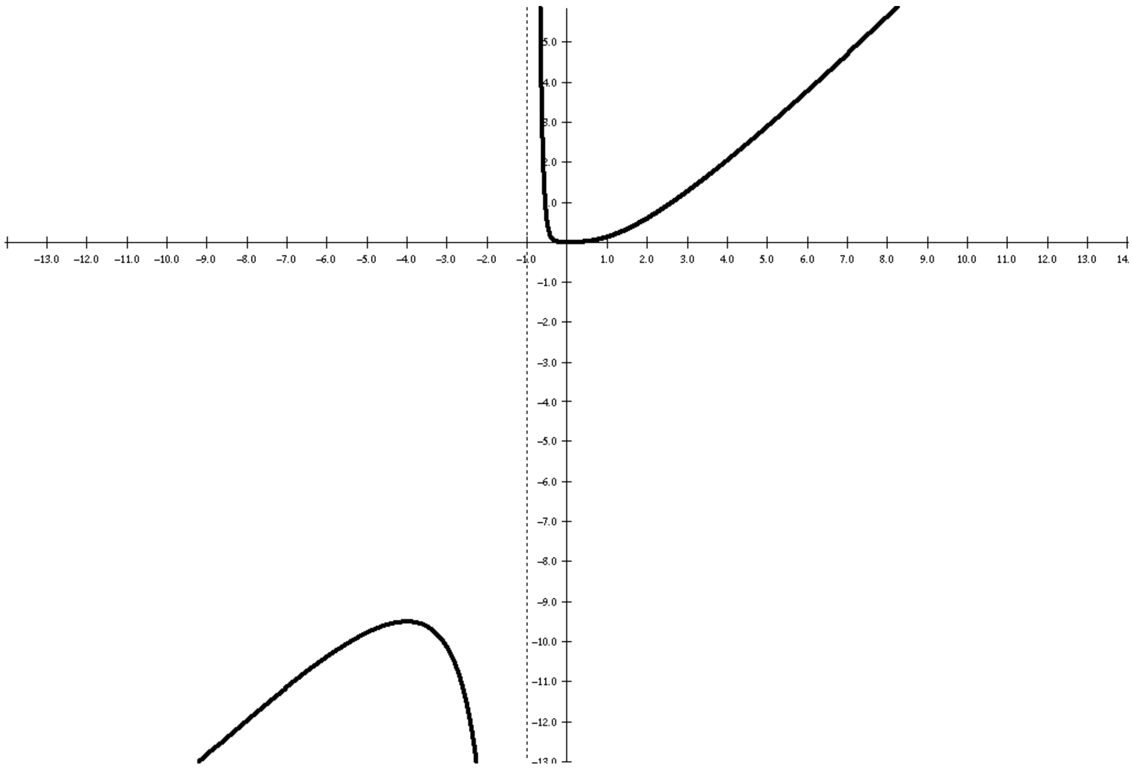
$\mathcal{D}(f) =$	$\mathbb{R}$
$f'(x) =$	$\frac{2x+1}{(x^2+1)^{\frac{3}{2}}}$
$f''(x) =$	$\frac{-4x^2-3x+2}{(x^2+1)^{\frac{5}{2}}}$
határértékek:	$\lim_{-\infty} f = -1$, $\lim_{+\infty} f = 1$
monoton nő:	$\left(-\frac{1}{2}, +\infty\right)$
monoton fogy:	$\left(-\infty, -\frac{1}{2}\right)$
szélsőérték:	$x = -\frac{1}{2}$ abszolút minimum
konvex:	$\left(\frac{-3-\sqrt{41}}{8}, \frac{-3+\sqrt{41}}{8}\right)$
konkáv:	$\left(-\infty, \frac{-3-\sqrt{41}}{8}\right) \cup \left(\frac{-3+\sqrt{41}}{8}, +\infty\right)$
inflexiós pontok:	$x = \frac{-3-\sqrt{41}}{8}$, $x = \frac{-3+\sqrt{41}}{8}$
$\mathcal{R}(f) =$	$[-2,236; 1)$



4. ábra. 6.(d)

(e) $f(x) = \frac{x^4}{(x+1)^3}$

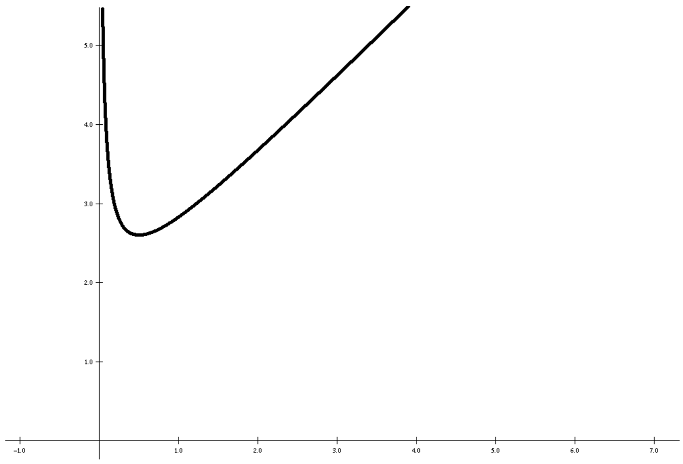
$\mathcal{D}(f) =$	$\mathbb{R} \setminus \{-1\}$
$f'(x) =$	$\frac{x^4+4x^3}{(x+1)^4}$
$f''(x) =$	$\frac{12x^2}{(x+1)^5}$
határértékek:	$\lim_{-1-} f = -\infty$, $\lim_{-1+} f = +\infty$,
	$\lim_{-\infty} f = -\infty$, $\lim_{+\infty} f = +\infty$
monoton nő:	$(-\infty, -4) \cup (0, +\infty)$
monoton fogy:	$(-4, -1) \cup (-1, 0)$
szélsőértékek:	$x = -4$ lokális maximum,
	$x = 0$ lokális minimum
konvex:	$(-1, +\infty)$
konkáv:	$(-\infty, -1)$
inflexiós pont:	–
$\mathcal{R}(f) =$	$(-\infty; -9,481] \cup [0, \infty)$



5. ábra. 6.(e)

(f) $f(x) = (1+x)^{\frac{3}{2}} \cdot x^{-\frac{1}{2}}$

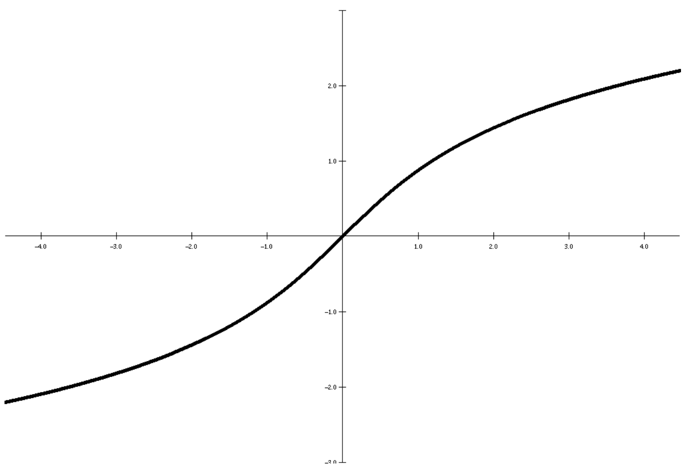
$\mathcal{D}(f) =$	$(0, +\infty)$
$f'(x) =$	$\frac{\sqrt{1+x}(2x-1)}{2x\sqrt{x}}$
$f''(x) =$	$\frac{6\sqrt{x}}{8x^3\sqrt{1+x}}$
határértékek:	$\lim_{0+} f = +\infty$, $\lim_{+\infty} f = +\infty$
monoton nő:	$\left(\frac{1}{2}, +\infty\right)$
monoton fogy:	$\left(0, \frac{1}{2}\right)$
szélsőérték:	$x = \frac{1}{2}$ abszolút minimum
konvex:	$(0, +\infty)$
konkáv:	–
inflexiós pont:	–
$\mathcal{R}(f) =$	$[2,598; \infty)$



6. ábra. 6.(f)

(g) $f(x) = \ln(x + \sqrt{x^2+1})$

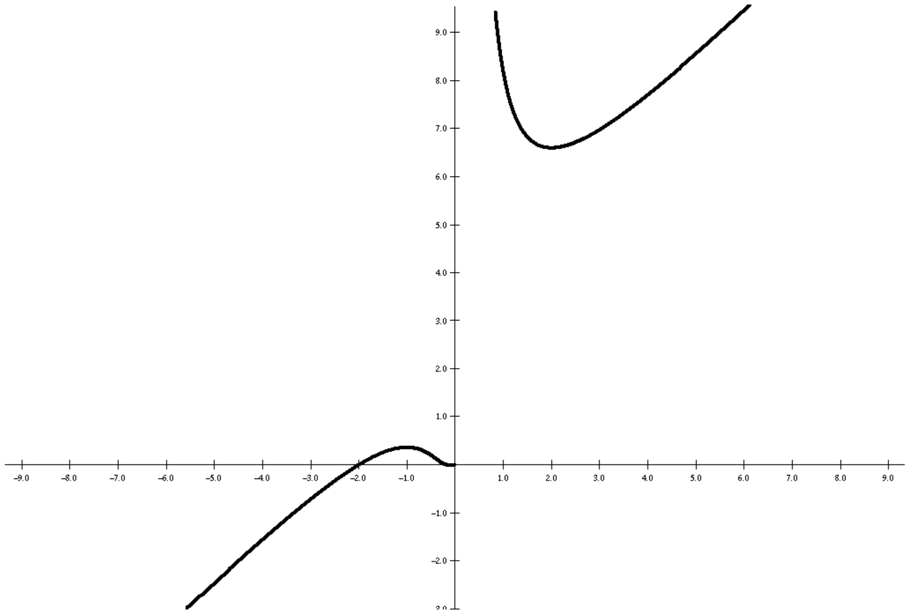
$\mathcal{D}(f) =$	$\mathbb{R}$
$f'(x) =$	$\frac{1}{\sqrt{x^2+1}}$
$f''(x) =$	$\frac{-x}{(x^2+1)^{\frac{3}{2}}}$
határértékek:	$\lim_{-\infty} f = -\infty$, $\lim_{+\infty} f = +\infty$
monoton nő:	$(-\infty, +\infty)$
monoton fogy:	–
szélsőérték:	–
konvex:	$(-\infty, 0)$
konkáv:	$(0, +\infty)$
inflexiós pont:	$x = 0$
$\mathcal{R}(f) =$	$\mathbb{R}$



7. ábra. 6.(g)

(h) $f(x) = (x+2) \cdot \mathrm{e}^{\frac{1}{x}}$

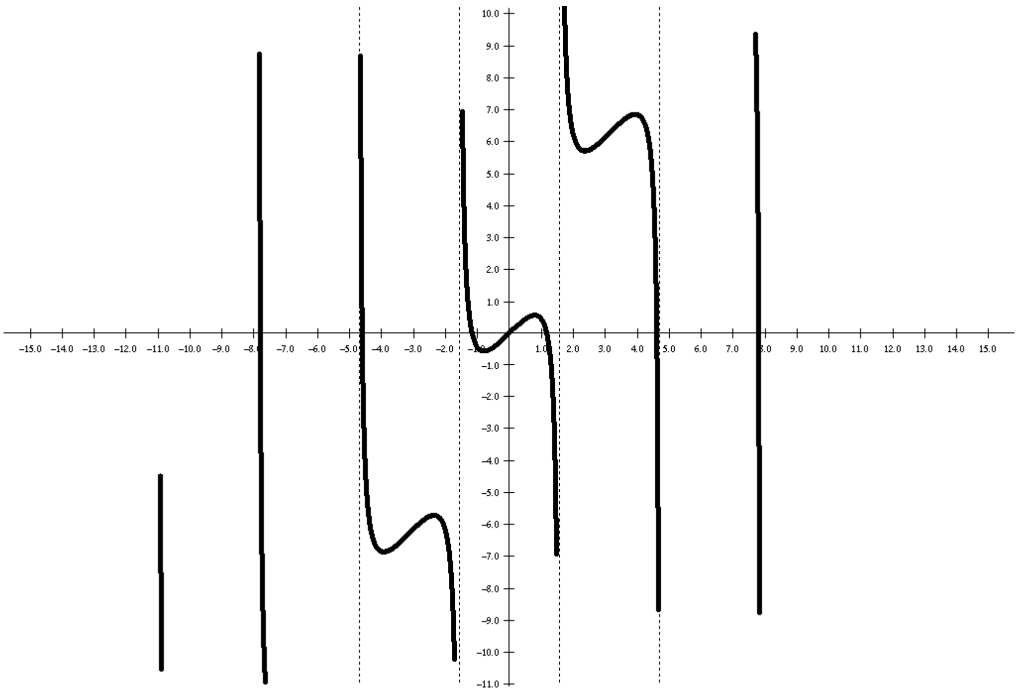
$\mathcal{D}(f) =$	$\mathbb{R} \setminus \{0\}$
$f'(x) =$	$\mathrm{e}^{\frac{1}{x}} \left(1 - \frac{1}{x} - \frac{2}{x^2}\right)$
$f''(x) =$	$\mathrm{e}^{\frac{1}{x}} \left(\frac{5}{x^3} + \frac{2}{x^4}\right)$
határértékek:	$\lim_{-\infty} f = -\infty$, $\lim_{+\infty} f = +\infty$
	$\lim_{0-} f = 0$, $\lim_{0+} f = +\infty$
monoton nő:	$(-\infty, -1) \cup (2, +\infty)$
monoton fogy:	$(-1, 0) \cup (0, 2)$
szélsőértékek:	$x = -1$ lokális maximum,
	$x = 2$ lokális minumum
konvex:	$\left(-\frac{2}{5}, 0\right) \cup (0, +\infty)$
konkáv:	$\left(-\infty, -\frac{2}{5}\right)$
inflexiós pont:	$x = -\frac{2}{5}$
$\mathcal{R}(f) =$	$(-\infty, 1/\mathrm{e}] \cup [4\sqrt{\mathrm{e}}, \infty)$



8. ábra. 6.(h)

(i) $f(x) = 2x - \operatorname{tg} x$

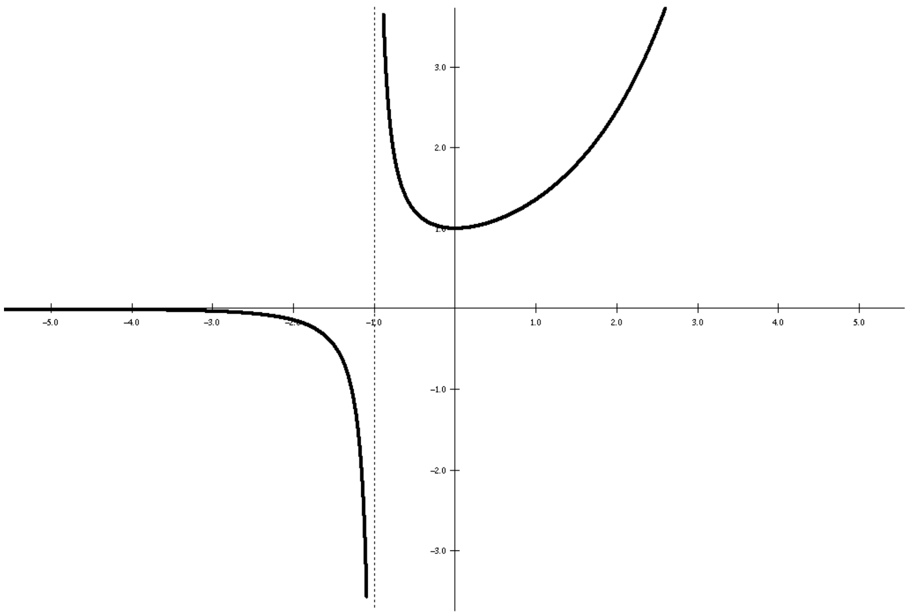
$\mathcal{D}(f) =$	$\mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\}$
$f'(x) =$	$2 - \frac{1}{\cos^2 x}$
$f''(x) =$	$-\frac{2\sin x}{\cos^3 x}$
határértékek:	$\lim_{\frac{\pi}{2}+k\pi-} f = -\infty$, $\lim_{\frac{\pi}{2}+k\pi+} f = +\infty$, $k \in \mathbb{Z}$
monoton nő:	$\left(-\frac{\pi}{4} + k\pi, \frac{\pi}{4} + k\pi\right)$, $k \in \mathbb{Z}$
monoton fogy:	$\left(\frac{\pi}{4} + k\pi, \frac{\pi}{2} + k\pi\right) \cup \left(\frac{\pi}{2} + k\pi, \frac{3\pi}{4} + k\pi\right)$, $k \in \mathbb{Z}$
szélsőértékek:	$x = -\frac{\pi}{4} + k\pi$, $k \in \mathbb{Z}$ lokális minimum,
	$x = \frac{\pi}{4} + k\pi$, $k \in \mathbb{Z}$ lokális maximum
konvex:	$\left(\frac{\pi}{2} + k\pi, \pi + k\pi\right)$, $k \in \mathbb{Z}$
konkáv:	$(k\pi, \pi + k\pi)$, $k \in \mathbb{Z}$
inflexiós pont:	$k\pi$, $k \in \mathbb{Z}$
$\mathcal{R}(f) =$	$\mathbb{R}$



9. ábra. 6.(i)

(j) $f(x) = \frac{e^x}{1+x}$

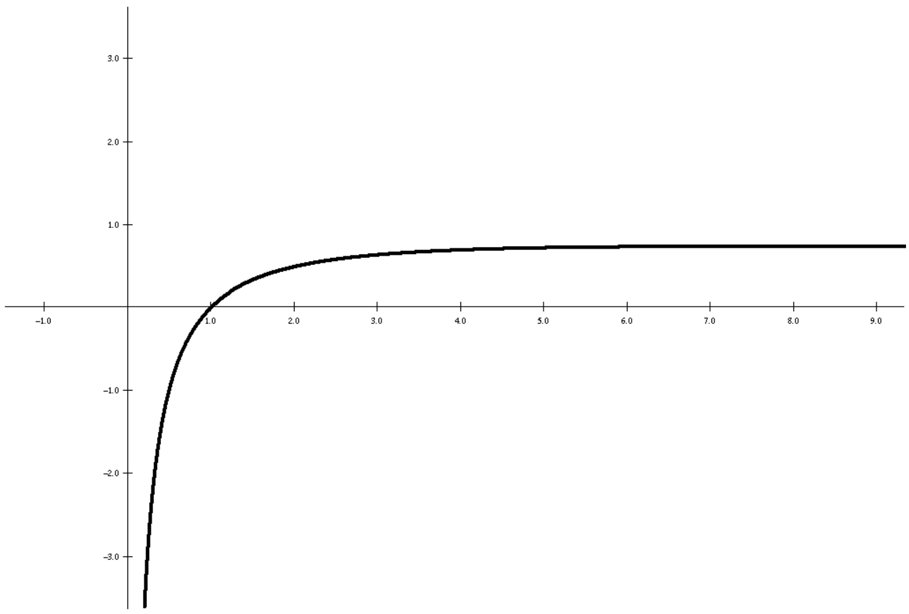
$\mathcal{D}(f) =$	$\mathbb{R} \setminus \{-1\}$
$f'(x) =$	$\frac{e^x x}{(1+x)^2}$
$f''(x) =$	$\frac{e^x\left(1+x^2\right)}{(1+x)^3}$
határértékek:	$\lim_{-\infty} f = 0$, $\lim_{+\infty} f = +\infty$,
	$\lim_{-1-} f = -\infty$, $\lim_{-1+} f = +\infty$
monoton nő:	$(0, +\infty)$
monoton fogy:	$(-\infty, -1) \cup (-1, 0)$
szélsőérték:	$x = 0$ lokális minimum
konvex:	$(-1, +\infty)$
konkáv:	$(-\infty, -1)$
inflexiós pont:	–
$\mathcal{R}(f) =$	$(-\infty, 0) \cup [1, \infty)$



10. ábra. 6.(j)

(k) $f(x) = \frac{\ln x}{\sqrt{x}}$

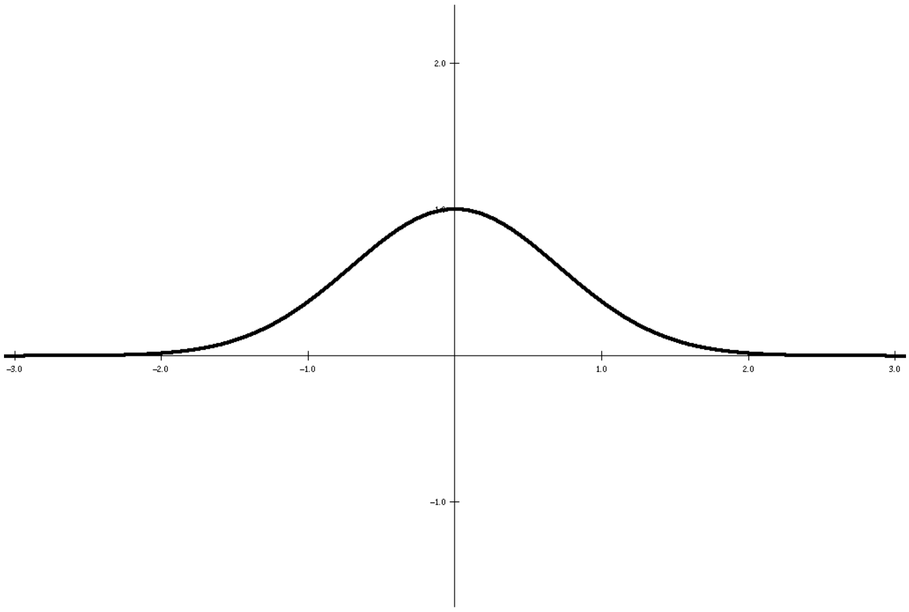
$\mathcal{D}(f) =$	$(0, +\infty)$
$f'(x) =$	$\frac{2-\ln x}{2x\sqrt{x}}$
$f''(x) =$	$\frac{-8+3\ln x}{4x^2\sqrt{x}}$
határértékek:	$\lim_{0+} f = -\infty$, $\lim_{+\infty} f = 0$
monoton nő:	$(0, \mathrm{e}^2)$
monoton fogy:	$(\mathrm{e}^2, +\infty)$
szélsőérték:	$x = \mathrm{e}^2$ abszolút maximum
konvex:	$\left(\mathrm{e}^{\frac{8}{3}}, +\infty\right)$
konkáv:	$\left(0, \mathrm{e}^{\frac{8}{3}}\right)$
inflexiós pont:	$x = \mathrm{e}^{\frac{8}{3}}$
$\mathcal{R}(f) =$	$(-\infty, \frac{2}{\mathrm{e}}]$



11. ábra. 6.(k)

(l) $f(x) = \mathrm{e}^{-x^2}$

$\mathcal{D}(f) =$	$\mathbb{R}$
$f'(x) =$	$-2x\mathrm{e}^{-x^2}$
$f''(x) =$	$\mathrm{e}^{-x^2}(4x^2-2)$
határértékek:	$\lim_{-\infty} f = \lim_{+\infty} f = 0$
monoton nő:	$(-\infty, 0)$
monoton fogy:	$(0, +\infty)$
szélsőérték:	$x = 0$ abszolút maximum
konvex:	$(-\infty, -\frac{1}{\sqrt{2}}) \cup (\frac{1}{\sqrt{2}}, +\infty)$
konkáv:	$(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$
inflexiós pontok:	$x = -\frac{1}{\sqrt{2}}$, $x = \frac{1}{\sqrt{2}}$
$\mathcal{R}(f) =$	$(0, 1]$



12. ábra. 6.(l)